Compressive Depth Map Acquisition Using a Single Photon-Counting Detector: Parametric Signal Processing Meets Sparsity

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Abstract

Active range acquisition systems such as light detection and ranging (LIDAR) and time-of-flight (TOF) cameras achieve high depth resolution but suffer from poor spatial resolution. In this paper we introduce a new range acquisition architecture that does not rely on scene raster scanning as in LIDAR or on a two-dimensional array of sensors as used in TOF cameras. Instead, we achieve spatial resolution through patterned sensing of the scene using a digital micromirror device (DMD) array. Our depth map reconstruction uses parametric signal modeling to recover the set of distinct depth ranges present in the scene. Then, using a convex program that exploits the sparsity of the Laplacian of the depth map, we recover the spatial content at the estimated depth ranges. In our experiments we acquired 64 × 64-pixel depth maps of fronto-parallel scenes at ranges up to 2.1 m using a pulsed laser, a DMD array and a single photon-counting detector. We also demonstrated imaging in the presence of unknown partially-transmissive occluders. The prototype and results provide promising directions for non-scanning, low-complexity range acquisition devices for various computer vision applications.

1. Introduction

Acquiring 3D scene structure is an integral part of many applications in computer vision, ranging from surveillance and robotics to human-machine interfaces. While 2D imaging is a mature technology, 3D acquisition techniques have room for significant improvements in spatial resolution, range accuracy, and cost effectiveness. Computer vision techniques—including structured-light scanning, depth-from-focus, depth-from-shape, and depth-from-motion [7,17,21]—are computation intensive, and the range output from these methods is highly prone to errors from miscalibration, absence of sufficient scene texture, and low signal-to-noise ratio (SNR) [13,17,20]. In comparison, active range acquisition systems such as LIDAR systems [19] and TOF cameras [6,8] are more robust against noise [13], work in real time at video frame rates, and acquire range information from a single viewpoint with little dependence on scene reflectance or texture. Both LIDAR and TOF cameras operate by measuring the time difference of arrival between a transmitted pulse and the scene reflection. Spatial resolution in LIDAR systems is obtained through raster scanning using a mechanical 2D laser scanning unit and light detection using a photon counting device such as an avalanche photodiode (APD) [1,3,15,19]. TOF cameras use a 2D array of range sensing pixels to acquire the depth map of a scene [6,8,16]. The scanning time limits spatial resolution in a LIDAR system, and due to limitations in the 2D TOF sensor array fabrication process and readout rates, the number of pixels in TOF camera sensors is also currently limited to a maximum of 320 × 240 pixels [16,18]. Moreover, in TOF-based range sensors, depth resolution is governed by
the pulse width of the source. Consequently, it is desirable to develop active range sensors with high spatial resolution without increasing the device cost and complexity.

In this paper, we introduce a framework for acquiring the depth map of a fronto-parallel scene using only a single time-resolved detector and a pulsed laser diode as the illumination unit. In our imaging setup (see Fig. 1), an omnidirectional, pulsed periodic light source illuminates the scene. Light reflected from the scene is focused onto a DMD which is then focused onto a single photon-counting APD that generates an intensity histogram of photon arrivals. This measurement process is repeated for a series of pseudorandomly-chosen binary DMD patterns. The recorded timing histograms are computationally processed to obtain a 2D scene depth map at the same pixel resolution as the DMD patterns.

2. Contributions

1. We demonstrate that it is possible to acquire a 2D depth map of a scene using a single time-resolved detector and no scanning components, with spatial resolution governed by the pixel resolution of the DMD array and the number of sensing patterns.

2. The parametric signal processing used in our computational depth map reconstruction achieves significantly better range resolution than conventional non-parametric techniques with the same pulse widths.

3. The use of sparsity-enforcing regularization allows trade-offs between desired spatial resolution and the number of sensing patterns.

4. We experimentally demonstrate our depth acquisition technique for typical scene ranges and object sizes using a low-power, near-infrared pulsed laser and an APD. We also demonstrate the effectiveness of our technique by imaging objects hidden behind partially-transmitting occluders, without any prior knowledge about the occluder.

3. Related Work

In this section we discuss challenges and prior work related to the use of compressive acquisition methods in TOF sensing.

3.1. Compressive Acquisition of Scene Reflectance

Many natural signals can be represented or approximated well using a small number of nonzero parameters. This property is known as sparsity and has been widely exploited for signal estimation and compression [5]. Making changes in signal acquisition architectures—often including some form of randomization—inspired by the ability to effectively exploit sparsity in estimation has been termed compressed sensing (CS). CS provides techniques to estimate a signal vector $x$ from linear measurements of the form $y = Ax + w$, where $w$ is additive noise and vector $y$ has fewer entries than $x$. The estimation methods exploit that there is a linear transformation $T$ such that $Tx$ is approximately sparse. An early instantiation of CS in an imaging context was the “single-pixel camera” [4, 23].

3.2. Challenges in Exploiting Sparsity in Range Acquisition

The majority of range sensing techniques attempt to acquire the entire signal corresponding to time-of-flight measurements either through raster scanning every point of interest in the field of view or establishing a correspondence between each spatial point and an array of sensors. The underlying signal of interest—depth—in these cases is naturally sparse in a wavelet domain or has sparse gradient or Laplacian. Furthermore, the depth map of a scene is generally more compressible or sparse than the reflectance or texture. Thus, we expect a smaller number of measurements to suffice; as expounded in Section 6, our number of measurements is 12% of the number of pixels as compared to 40% for reflectance imaging [4, 23].

Compressively acquiring range information using only a single detector poses two major challenges. First, the quantity of interest—depth—is embedded in the reflected signal as a time shift. The measured signal at the detector is a sum of all reflected returns and hence does not directly measure this time shift. This nonlinearity worsens when there are multiple time shifts in the returned signal corresponding to the presence of many depths. The second challenge comes from the fact that a single detector loses all directionality information about the reflected signals; this is present in reflectance imaging as well.

3.3. Compressive LIDAR

In a preliminary application of the CS framework to range acquisition in LIDAR systems [11], 2 ns square pulses from a function generator drive a 780 nm laser diode to illuminate a scene. Reflected light is focused onto a DMD that implements pseudorandomly-chosen binary patterns. Light from the sensing patterns is received at a photon-counting detector and gated to collect photons arriving from an a priori chosen range interval, and then conventional CS reconstruction is applied to recover an image of the objects within the selected depth interval. The use of impulsive illumination and range gating make this a conventional CS problem in that the quantities of interest (reflectances as a function of spatial position, within a depth range) are combined linearly in the measurements. Hence, while this approach unmixes spatial correspondences it does
not directly solve the aforementioned challenge of resolving nonlinearly-embedded depth information. The need for accurate range intervals of interest prior to reconstruction is one of the major disadvantages of this system. It also follows that there is no method to distinguish between objects at different depths within a chosen range interval. Moreover, acquiring a complete scene depth map requires a full range sweep. The proof-of-concept system [11, 12] has 60 cm range resolution and 64 × 64 pixel resolution.

3.4. Compressive Depth Acquisition Camera

Resolving both the challenges with compressive depth sensing was first demonstrated in [14] in the near-range case for small objects. Their demonstration considered near-range scenes—less than 1 ft away from the imaging setup. The imaging device uses patterned illumination and a single sensor to record reflected observations. Additionally, the framework considers only piecewise planar, Lambertian objects with simple shapes and does not address transmissive occlusions.

4. Signal Modeling of Scene Response and Depth Map Reconstruction

Consider a piecewise-planar scene comprising two fronto-parallel objects as shown in Fig. 1.

4.1. Parametric Response of Fronto-Parallel Facets

When an omnidirectional pulsed source illuminates the scene, the signal \( r(t) \) received at the single time-resolved photodetector is the convolution of a parametric signal \( p(t) \) with the pulse shape \( s(t) \). The parametric signal \( p(t) \) comprises time-shifted returns corresponding to the objects at depths \( d_1 \) and \( d_2 \). The returns from each object are highly concentrated in time because the scene is in far-field and the object dimensions are small compared to the distances \( d_1 \) and \( d_2 \). For complete scene response modeling we refer the reader to [14]. Note that the parametric signal \( p(t) \) is completely characterized by four parameters: the locations and amplitudes of the two peaks. The problem of recovering these signal parameters from the discrete samples of \( r(t) \), a lowpass filtered version of \( p(t) \), is a canonical finite rate of innovation (FRI) sampling problem [22]. Note that the parameters of the scene response \( p(t) \) only furnish information about the depth ranges present in the scene; they convey no information about the object shapes and their positions in the field-of-view of the imaging device.

4.2. Shape and Transverse Position Recovery

The next step is to obtain the shapes of objects and their transverse positions in the depth map. A single patterned return only provides depth information. However, when repeated for multiple pseudorandomly-chosen binary patterns we find that the the heights of the peaks in the returned signal contribute useful information that help identify object shape. Note that the depth map \( D \) is a weighted combination of the two depth masks \( I^1 \) and \( I^2 \), i.e., \( D = d_1 I^1 + d_2 I^2 \) [14]. Each binary-valued depth mask identifies the positions in the scene where the associated depth is present, thereby identifying the shape and transverse position of the object present at that depth. Having estimated \( d_1 \) and \( d_2 \) using parametric recovery of the signal \( p(t) \), the problem of estimating \( I^1 \) and \( I^2 \) is a linear inverse problem. This is because the amplitude of the signal at the time instances corresponding to depths \( d_1 \) and \( d_2 \) is equal to the inner product of the DMD pattern \( C \) with the depth masks \( I^1 \) and \( I^2 \) respectively. Furthermore, the assumption that the scene is fronto-parallel translates to the Laplacian of the depth map being sparse. Hence, we may possibly recover \( I^1 \) and \( I^2 \) using far fewer patterns, \( M \), than the number of pixels \( N^2 \).

For each pattern \( C_i, i = 1, \ldots, M \), the digital samples of the received signal, \( r_i[n] \), are processed using the parametric deconvolution framework to obtain the amplitudes of the recovered parametric signals \( p_i[n] \) corresponding to the inner products \( y^1_i = \langle C_i, I^1 \rangle \) and \( y^2_i = \langle C_i, I^2 \rangle \). This data can be compactly represented using the linear system

\[
[y^1_i \ y^2_i] = C_{i\times M} \begin{bmatrix} \text{vec}(I^1) \\ \text{vec}(I^2) \end{bmatrix}.
\]

This is an underdetermined system of linear equations because \( M \ll N^2 \). But, since the Laplacian of the depth map \( D \) is sparse, we can potentially solve for good estimates of the depth masks \( I^1 \) and \( I^2 \) using the sparsity-enforcing joint optimization framework outlined in the next section.

5. Depth Map Reconstruction

We propose the following optimization program for recovering the depth masks \( I^1 \) and \( I^2 \) and hence, the depth map \( D \) from the observations \( \begin{bmatrix} y^1 \ y^2 \end{bmatrix} \):

\[
\text{OPT:} \min_D \| y^1 - C \text{vec}(I^1) \|_F^2 + \| (\Phi \otimes \Phi^T) D \|_1
\]

subject to \( I^1_{k\ell} + I^1_{k\ell} + I^2_{k\ell} = 1, \) for all \((k, \ell)\),

\[
D = d_1 I^1 + d_2 I^2,
\]

and \( I^1_{k\ell}, I^2_{k\ell} \in \{0, 1\}, \) \( k, \ell = 1, \ldots, N \).

Here the Frobenius matrix norm squared \( \| \cdot \|_F^2 \) is the sum-of-squares of the matrix entries, the matrix \( \Phi \) is the second-order finite difference operator matrix

\[
\Phi = \begin{bmatrix}
1 & -2 & 1 & 0 & \cdots & 0 \\
0 & 1 & -2 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & 1 & -2 & 1
\end{bmatrix},
\]
Figure 2. Depth map reconstruction algorithm. The intensity samples \( r_i[n] \) acquired for each binary pattern \( C_i \) are first processed using parametric deconvolution to recover the scene response \( p_i[n] \). The positions of peak amplitudes in \( p_i[n] \) provide depth estimates \( d_1 \) and \( d_2 \) while the amplitudes themselves are used to recover the spatial structure of the depth map (i.e. the depth masks \( I^1 \) and \( I^2 \)) at these depth locations. The spatial resolution recovery is accomplished using a convex program that enforces gradient-domain sparsity and also includes a robustness constraint.

The number of nonzero entries (the “\( \ell_0 \) pseudonorm”) is difficult to use because it is nonconvex and not robust to small perturbations, and the \( \ell_1 \) norm is a suitable proxy with many optimality properties [5]. The problem OPT combines the above objective with maintaining fidelity with the measured data by keeping \( \|y_1 - y_2 - C[\text{vec}(I^1) - \text{vec}(I^2)]\|_F^2 \) small. \( I_{k,l}^k \) is the depth mask corresponding to the portions of the scene that did not contribute to the returned signal. The constraints \( I_{k,l}^k, I_{k,l}^1, I_{k,l}^2 \in \{0, 1\} \) and \( I_{k,l}^0 + I_{k,l}^1 + I_{k,l}^2 = 1 \) for all \((k, l)\) are a mathematical rephrasing of the fact that each point in the depth map has a single depth value so different depth values cannot be assigned to one position \((k, l)\). The constraint \( \Phi D = d_1I^1 + d_2I^2 \) expresses how the depth map is constructed from the index maps. While the optimization problem OPT already contains a convex relaxation in its use of \( \|\Phi D\|_1 \), it is nevertheless computationally intractable because of the integrality constraints \( I_{k,l}^0, I_{k,l}^1, I_{k,l}^2 \in \{0, 1\} \). We further relax this constraint to \( I_{k,l}^0, I_{k,l}^1, I_{k,l}^2 \in [0, 1] \) to yield a tractable formulation. We also show in Section 6.1 that this relaxation allows us to account for partially-transmissive objects in our scenes. We solved the convex optimization problem with the relaxed integrality constraint using CVX, a package for specifying and solving convex programs [9, 10]. Note that this optimization framework solves a single CS problem and optimization without range gating. Techniques employed by [12] assume knowledge of ranges of interest \textit{a priori} and solve a CS-style optimization problem per range of interest. In the next section we discuss our experimental prototype and computational reconstruction results.

6. Setup, Data Acquisition, and Results

The imaging device as shown in Fig. 1 consists of an illumination unit and a single detector. The illumination unit comprises a function generator that produces 2 ns square pulses that drive a near-infrared (780 nm) laser diode to illuminate the scene with 2 ns Gaussian pulses with 50 mW peak power and a repetition rate of 10 MHz. Note that the pulse duration is shorter than the repetition rate of the pulses. The detector is a cooled APD operating in Geiger mode. When a single photon is absorbed, the detector outputs a TTL pulse about 10 ns in width, with edge timing resolution of about 300 ps. After a photon is absorbed, the detector then enters a dead time of about 30 ns during which it is unable to detect photons. To build the histogram of arrival times, we use a correlating device (Picoquant Time-harp) designed for time-correlated single-photon counting. The correlator has two inputs: \textit{start} and \textit{stop}. The output of the laser pulse generator is wired to start, and the APD output is wired to stop. The device then measures differences in arrival times between these two inputs to build up timing histograms over an acquisition time \( t_a \); this acquisition time was different for the two scenes in our experiment. For each histogram the scene receives \( R \) pulses where \( R = \text{Repetition rate (10 MHz)} \times t_a \). The photon counting mechanism and the process of building the timing histogram are shown in Fig. 4.

Scenes are set up so that objects are placed fronto-parallely between 0.3 m to 2.8 m from the device. Objects are 1 ft\(^2\) cardboard cut-outs of the letters U and R at distances \( d_1 \) and \( d_2 \) respectively. Light reflected by the scene is imaged onto a DMD through a 10 nm filter centered at 780 nm with a 38 mm lens focused to infinity with respect to the DMD. We use a 0.55 inch D4100 Texas Instru-
Imaging scenes with unknown transmissive occluders.

In the second scene we consider a combination of transmissive and opaque objects and attempt to recover a depth map. The scene is shown in Fig. 6 a. Note that the burlap placed at 1.4 m from the imaging device completely fills the field of view. A 2D image of the scene reveals only the burlap. However, located at 2.1 m from the imaging device are cardboard cut-outs of U and R—both at the same depth. These objects are completely occluded in the 2D reflectance image. Also seen in Fig. 6 is a timing histogram acquired with acquisition time \( t_a = 4 \text{s} \). The histogram shows that the burlap contributes a much larger reflected signal (12 times stronger) than the contribution of the occluded objects. Figs. 6 b, c show depth masks \( I^1 \), \( I^2 \) for the burlap and occluded objects respectively for 500 patterns while Figs. 6 d, e show depth masks obtained using 2000 patterns. The reconstruction of the depth map in the presence of a transmissive occluder is possible because of the relaxation of the integrality constraint.
Figure 5. Reconstruction results. (a) Scene setup. (b), (c), and (d) Depth masks $I^1$ and $I^2$ and the background mask $I^0$ reconstructed using 500 patterns. (e) and (f) Depth masks reconstructed using 2000 patterns.

Figure 7. Depth map reconstruction with simulated scene response to longer source pulse width. We simulate poor temporal resolution by lowpass filtering the captured intensity profiles so that the Gaussian pulses overlap and interfere with the signal amplitudes. The effectiveness of the parametric deconvolution technique is demonstrated by accurate recovery of the depth map as shown in the reconstructed depth map.

High range resolution with slower detectors. When planar facets are separated by distances that correspond to time differences greater than the pulse width of the source, the time shift information can be trivially separated. The more challenging case is when facets are closely spaced or there are large number of distinct facets. A detailed analysis of these cases for recovering depth information can be found in [14]. However, in this paper we focus on well-separated fronto-parallel planar facets. We briefly address the case where our scenes are illuminated by a system with longer source pulse width. This results in time shift information from planar facets bleeding into each other, that is, peaks that are not well separated in the returned signal. The use of a parametric signal modeling and recovery framework [22] enables us to achieve high depth resolution relative to the speed of the time sampling at the photodetector. We demonstrate this through simulating longer source pulse width by smearing the timing histograms to correspond to a source four times as slow as the source in the experiments.

Additionally, we address the case of recovering depth information when the pulse width of the source is longer than the time-difference corresponding to the minimum distance between objects. Techniques such as those implemented in [12] will fail to resolve depth values from a returned signal that suffers interference of information between nearby objects. Achieving range resolution higher than that possible with inherent source bandwidth limitations is an important contribution made possible by the parametric recovery process introduced in this work.

7. Discussion and Conclusions

We have described a depth acquisition system that can be easily and compactly assembled with off-the-shelf components. It uses parametric signal processing to estimate range information followed by a sparsity-enforcing reconstruction to recover the spatial structure of the depth map.

Acquiring an $N \times N$-pixel depth map by raster scanning ordinarily would take $N^2$ measurements as in LIDAR techniques or $N^2$ sensors in a time-of-flight camera. Our framework shows that measurements (or patterns), $M$, as low as 12% of the total number of pixels, $N^2$, provide a reasonable reconstruction of a depth map. This is achieved by modeling the reflected scene response as a parametric signal with a finite rate of innovation [22] and combining this with compressed sensing-style reconstruction. Existing TOF cameras and LIDAR techniques do not use the sparsity inherent in scene structure to achieve savings in number of sensors or scanning pixels.

We also achieve high range resolution by obtaining depth information through parametric deconvolution of the returned signal. In comparison LIDAR and TOF that do not leverage the parametric nature of the reflected signal are limited in range resolution by inherent source pulse width, i.e., the use of a longer pulse width would make it infeasible to recover depth information and hence spatial information correctly. The compressive LIDAR framework in [12] is also limited in range resolution by the source-detector bandwidths, that is, the use of a source with longer pulse width would make it challenging to resolve depth information and hence spatial correspondences correctly.

The processing framework introduced in this paper solves a single optimization problem to reconstruct depth
In contrast the system demonstrated in [12] relies on gating the returned signals in a priori known range intervals and hence solves as many optimization problems as there are depths of interest in the scene. Consequently, direct limitations are lack of scalability in the presence of increasing depth values and inaccuracies introduced by insufficient knowledge of range intervals. Additionally, the robustness constraint used in our optimization problem is also key to jointly reconstructing the depth map using a single optimization problem to recover a depth map with a smaller number of patterns.

Our experiments acquire depth maps of real-world scenes in terms of object sizes and distances. The work presented in [14] focused on objects of smaller dimensions (less than 10 cm) and at shorter ranges (less than 20 cm). The experiments in this paper are conducted at longer ranges (up to 2.1 m from the imaging device) with no assumptions on scene reflectivity and more importantly at low light levels. We also address the case when transmissive occluders are present in the scene. In [14] illumination patterns were projected on to the scene with a spatial light modulator. When these patterns are projected at longer distances they suffer distortions arising from interference. The setup described in this paper uses patterns at the detector thereby implicitly resolving the aforementioned challenge in patterned illumination.

Future extensions of the work presented in this paper include information theoretic analysis of range resolution in photon-limited scenarios and range resolution dependence on the number and size of time bins. We also intend to investigate the number of sensing patterns required to achieve a desired spatial resolution. On the experimental side, our future work involves higher pixel-resolution depth map acquisitions of complex natural scenes.

References